I workshop Dyna.M.I.Ch.E – 2024
Dynamical Methods: Inverse problems, Chaos and Evolution

Palermo, March 22, 2024, at 3:00 pm, UTC+01:00

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Day's Program

- 15:00 Opening by
  - **Luca Zampogni** (Principal Investigator of the Gnampa Project 2024 Dyna.M.I.Ch.E),
  - **Roberto Livrea** (Head of the Department of Mathematics and Computer Sciences - University of Palermo)
  - **Gianluca Vinti** (Head of the "Centro di Ricerca Interdipartimentale Lamberto Cesari")

Chairman: **Luisa di Piazza**

- 15:30-16:05 **Bianca Satco**, *On the viability of differential inclusions involving Steltjes derivative*
- 16:10-16:45 **Vladislav Kravchenko**, *Neumann series of Bessel functions representations and reconstruction techniques in inverse coefficient problems*
- 16:50-17:25 **Paulo Varandas**, *Historic behavior in linear dynamics*
- 17:30-17:55 **Michele Piconi**, *Regularization results for Durrmeyer sampling type operators in L^p-spaces*
- 18:00- 18:25 **Lorenzo Boccali**, *Approximation by sampling Kantorovich operators of max-product type in functional spaces*
- 18:30 Closing of the meeting

On the viability of differential inclusions involving Steltjes derivative

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Keywords. *Viability; Differential inclusion; Stieltjes derivative.*

The topic of this talk is a viability result proved in a joint work with G. Smyrlis ([5]): for a Cauchy set-valued problem involving the Stieltjes derivative ([3]) with respect to a left-continuous non-decreasing function $g : [0, 1] \to \mathbb{R}$

$$x'_g(t) \in F(t, x(t)),\ \mu_g - a.e.\ t \in [0, 1), \quad x(0) = x_0$$

we study the existence of g-absolutely continuous ([2]) solutions which, starting from $x_0 \in K(0)$, keep this feature on the whole unit interval:

$$x(t) \in K(t),\ \text{for every } t \in [0, 1].$$

By μ_g one denotes the Lebesgue-Stieltjes measure induced by the map g.
The map $K : [0, 1] \to \mathcal{P}_{cc}(\mathbb{R}^d)$ is g-absolutely continuous from the left w.r.t. the topology generated by g ([2]), while $F : [0, 1] \times \mathbb{R}^d \to \mathcal{P}_{cc}(\mathbb{R}^d)$ is assumed to be upper semicontinuous for the product of the g-topology of $[0, 1]$ with the usual topology of $\mathbb{R}^d$.
As in the classical case, a tangential condition is imposed, using a generalization of the notion of contingent derivative, namely the contingent g-derivative ([4]).
Classical viability results (for usual differential inclusions, [1]) are thus generalized and, at the same time, the gate to new viability results for difference inclusions, impulsive differential inclusions or dynamic inclusions on time scales is open.

References

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Neumann series of Bessel functions representations and reconstruction techniques in inverse coefficient problems

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Keywords. *Neumann series, Bessel functions representation, inverse coefficient problems, Sturm-Liouville equation.*

Functional series representations for solutions of linear differential equations encounter numerous applications in solving direct spectral, scattering and other types of problems. In the talk the recently discovered Neumann series of Bessel functions representations for solutions of Sturm-Liouville equations with complex valued coefficients are presented (see the monographs [1], [2] and references therein), their convergence properties and applications are discussed. These properties make the representations especially convenient for solving inverse coefficient problems. The corresponding approach is the main object of the talk. We show that difficult and numerically challenging inverse problems are reduced to systems of linear algebraic equations for the coefficients of the representations, and the Sturm-Liouville equation is recovered from the first coefficient.

In particular, we discuss a general inverse coefficient problem for a Sturm-Liouville equation with an unknown complex valued coefficient. Special cases of the problem include the recovery of the potential from a Weyl function, the inverse two-spectra Sturm-Liouville problem, the inverse scattering problem and the inverse transmission eigenvalues problem among others. A variety of inverse coefficient problems for partial differential equations and for quantum graphs are also reduced to the considered problem. The approach leads to a simple and efficient numerical algorithm, that is illustrated by numerical examples.

References

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Historic behavior in linear dynamics

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Keywords. *Linear cocycles; Linear dynamics; Ergodic theorem; Historic behavior.*

The foundations of the classical ergodic theory rely on ergodic theorems, which guarantee an almost everywhere convergence of time averages (with respect to invariant probability measures). Notwithstanding, it is often the case that the set of points with historic behavior (that is for which the ergodic theorem fails) is quite relevant e.g. has full entropy, full Hausdorff dimension, full metric mean dimension, etc. In this talk I will discuss some mechanisms that give rise to the presence of (many) points with historic behavior in the context of random products of matrices. If time permits, I will also discuss further perspectives in the context of linear dynamics.

References

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Regularization results for Durrmeyer sampling type operators in L^p-spaces

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Keywords. *Durrmeyer sampling type operators* | *Regularization properties* | *Fourier transform* | *Bernstein classes* | *Convolution*

In [4], we start a study on the approximation properties of a semi-discrete version of sampling operators, specifically the *Durrmeyer sampling type operators* (DSO) [6, 2]. As it is well-known, the theory of sampling series, in one and several variables, holds a central position in the field of Approximation Theory due to its widespread applications, especially in Signal and Image Processing. Originating from a sharp modification of Bernstein polynomials, DSO play a crucial role in extending the celebrated Weierstrass approximation theorem to broader functional spaces, involving a wide spectrum of functions. Notably, DSO also expand other well-known families of sampling operators, including both generalized [3] and Kantorovich [1] types.

In this talk, we present a recent study on the regularization properties of DSO in L^p-spaces ($1 \leq p \leq +\infty$) using a distributional approach. We highlight how the regularization depends on the regularity of the discrete kernel φ. In [5], we explore cases where φ is continuous, in Sobolev spaces, and when it is bandlimited, providing a closed form for the distributional Fourier transform of these operators applied to bandlimited functions.

The talk concludes by discussing the main results through specific instances of bandlimited kernels, such as Fejér and Bochner-Riesz kernels.

References

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Approximation by sampling Kantorovich operators of max-product type in functional spaces

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Keywords. *Max-product sampling Kantorovich operators; Generalized kernels; Orlicz spaces; Modular convergence; Quantitative estimates.*

Beginning in the first half of the 1900s, with the classical Whittaker-Kotel'nikov-Shannon (WKS) sampling theorem, the study of sampling-type operators for signal approximation has experienced a constant increase in interest among the scientific community. Since the required assumptions on the signal to be approximated give rise to limitations, from an application point of view, of the above theorem, several approximate versions of the classical sampling series have been introduced and investigated from the 1960s until now for the reconstruction of both continuous and not-necessarily continuous functions.
Recently, in [4], the authors introduced the so-called max-product sampling Kantorovich operators K_n^{χ} based upon generalized kernels. The main approximation properties, including convergence results and quantitative estimates, for this non-linear family of sampling-type operators have been established in the continuous setting and in the L^p-spaces, $1 \leq p < +\infty$. The max-product version of sampling Kantorovich operators is constructed by computing the supremum (or maximum in the finite case), denoted by the symbol $\bigvee$, of a given set of real numbers and not the series (or sum in the case of finite terms), as in the definition of the linear version [1]. As it is known from the literature (see, e.g., [5, 6, 2]), several approximation operators of max-product type return a sharper approximation than their corresponding linear counterparts.
In this talk, we will show that, by adopting the so-called moment-type approach in its max-product variant to the kernels, considered in order to define the above operators, it is possible to prove a modular convergence theorem (see [3]) for K_n^{χ} when approximation processes in the setting of Orlicz spaces L^{φ} are considered on both bounded intervals and on the whole real axis. The latter result, which extends by a unique general ap-

proach those proved in [4], makes it possible to provide a unifying theory concerning the convergence properties of the max-product sampling Kantorovich operators in a wide range of functional spaces, including the interpolation and exponential spaces. Finally, we will present some specific examples of well-known kernels for K_n^χ, such as Fejér or B-spline-type kernel, for which the above theory can be applied.

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